

BACKPAPER EXAMINATION
B. MATH III YEAR, I SEMESTER 2011-2012
DIFFERENTIAL EQUATIONS

Max. 100.

Time limit: 3hrs

The five questions carry a total of 110 marks. Answer as many as you can.

1. Show that $\sum_{n=0}^{\infty} \frac{x^n}{(n!)(n+2)}$ is a solution of the equation $x^2 y'' + 2xy' - 2y = (x-1)e^x$ on \mathbb{R} . [10]

2. Find all solutions of the differential equation in above problem by guessing one solution of the corresponding homogeneous equation. [20]

3. Show that the hypergeometric function $F(a, b, c, z) = \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(n!)(c)_n} z^n$ (where $(\alpha)_k = \alpha(\alpha+1)\dots(\alpha+k-1)$) is a solution of the equation $z(1-z)y'' + [c - (a+b+1)z]y' - aby = 0$ on $\{z \in \mathbb{C} : |z| < 1\}$. [25]

4. Find all solutions of $\frac{\partial^2 u}{\partial x^2} + x \frac{\partial u}{\partial y} = \sin(y)$ that can be obtained by separation of variables. [25]

5. For the equation $4xy'' + (3-4x)y' - 4y = 0$ find two linearly independent solutions of the type $x^\alpha \sum_{n=0}^{\infty} a_n x^n$. [30]